Wind simulation using high-frequency velocity component measurements

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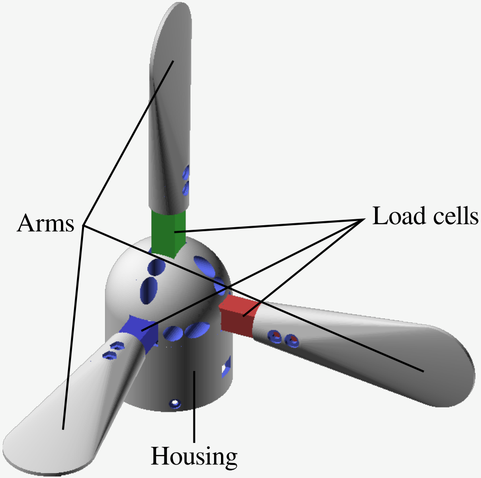
Motivation

- Datasets with all three wind velocity components and one-second resolution are difficult to find.
- Three-axis ultrasonic anemometers are expensive.
- We needed three-dimensional autocovariance function to simulate wind with autoregressive model.

Solution: make our own anemometer based on load cells.

Three-axis anemometer based on load cells



Load cell capacity	1 kg
Load cell amplifier	HX711
Microcontroller	ATmega328P

Calculating wind velocity

Bernoulli's equation:

$$\rho\frac{v^2}{2} + \rho g z = p_0 - p$$

Pressure force:

$$\vec{F} = pS\vec{n}.$$

Wind velocity:

$$\begin{cases} v_x^2 \propto F_x \\ v_y^2 \propto F_y \\ v_z^2 \propto F_z \end{cases} \Rightarrow \begin{cases} v_x = \alpha_x\sqrt{F_x} \\ v_y = \alpha_y\sqrt{F_y} \\ v_z = \alpha_z\sqrt{F_z} \end{cases}$$

Per-axis probability distribution function for wind velocity

Weibull distribution (scalar velocity)[1]:

$$f\left(v; b, c\right) = bc\left(bv\right)^{c-1} \exp\left(-\left(bv\right)^{c}\right).$$

Weibull distribution (per-axis scalar velocity):

$$f\left(v_x; b_1, c_1, b_2, c_2\right) = \begin{cases} b_1 c_1 \left(b_1 \left|v_x\right|\right)^{c_1 - 1} \exp\left(-\left(b_1 \left|v_x\right|\right)^{c_1}\right). & \text{if } v_x < 0 \\ b_2 c_2 \left(b_2 \left|v_x\right|\right)^{c_2 - 1} \exp\left(-\left(b_2 \left|v_x\right|\right)^{c_2}\right). & \text{if } v_x \geq 0 \end{cases}$$

Parameters:

- b scale parameter
- c shape parameter

[1] C. Justus, W. Hargraves, A. Yalcin *Nationwide Assessment of Potential Output from Wind-Powered Generators*, 1976.

Three-dimensional ACF of wind velocity

One-dimensional autocovariance function[2]:

$$K(t) = a_3 \exp\left(-\left(b_3 t\right)^{c_3}\right).$$

Three-dimensional autocovariance function:

$$K(t, x, y, z) = a \exp\left(-\left(b_t t\right)^{c_t} - \left(b_x x\right)^{c_x} - \left(b_y y\right)^{c_y} - \left(b_z z\right)^{c_z}\right).$$

[2] G. Box, G. Jenkins *Time series analysis: forecasting and control*, 1976.

Data collection and preprocessing

Calibration coefficients:

Axis	C_1	C_2
X	11.19	12.31
Y	11.46	11.25
Z	13.55	13.90

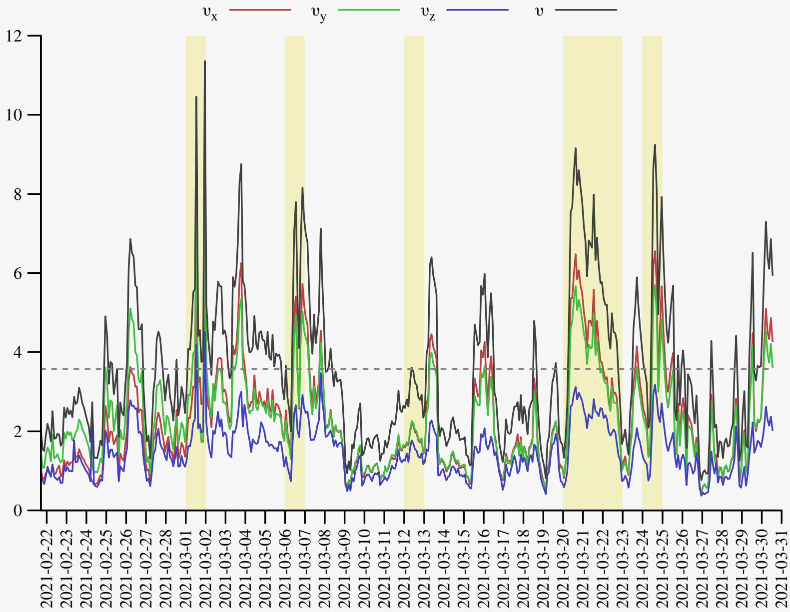
Dataset properties:

Time span	36 days
Size	122 Mb
No. of samples	3 157 234
No. of samples after filtering	2 775 387
Resolution	1 sample per second

C_1 — negative values, C_2 — positive values.

```
sampleToSpeed ← function(x, c1, c2) {
 t ← c(1:length(x))
 reg ← lm(x~t)
 x ← x - reg$fitted.values  # remove linear trend
 x ← sign(x)*sqrt(abs(x))   # convert from force to velocity
 x[x<0] = x[x<0] / c1       # scale sensor values to wind speed
 x[x>0] = x[x>0] / c2       # using calibration coefficients
 x }
```

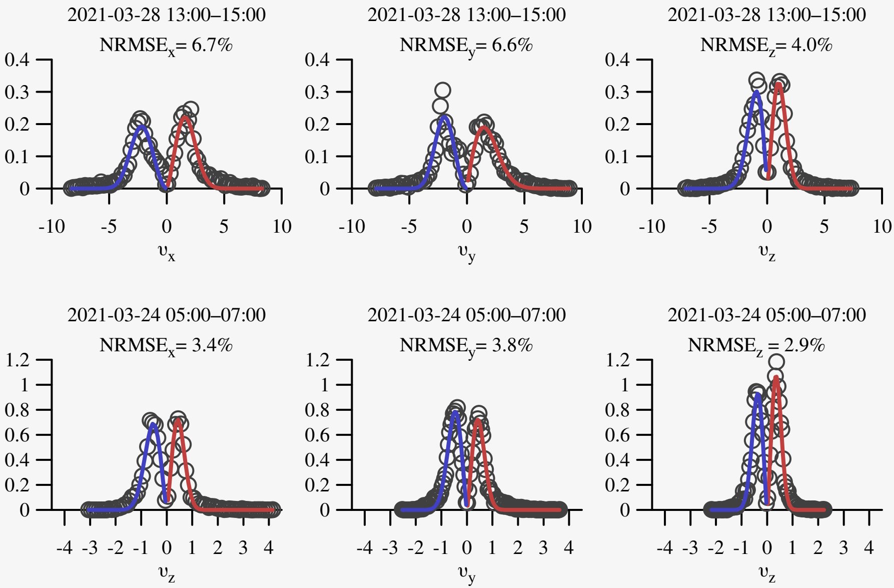
Verification against EMERCOM data



$\upsilon_x, \upsilon_y, \upsilon_z, \upsilon$ — mean velocities.

Yellow regions — wind velocity above average as reported by EMERCOM of Russia https://en.mchs.gov.ru/.

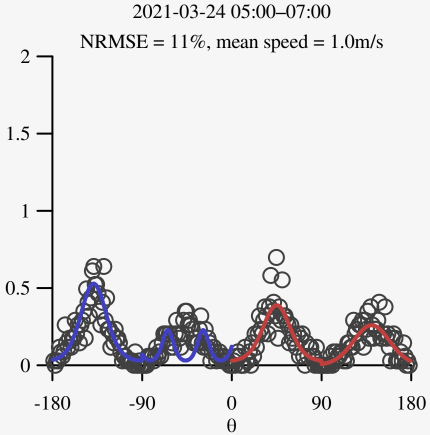
Verification of wind velocity against Weibull distribution



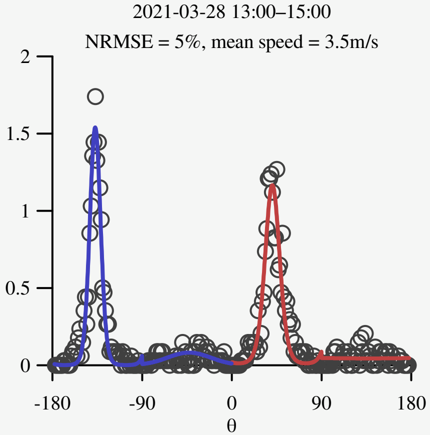
largest error

smallest error

Verification of wind direction against von Mises distribution



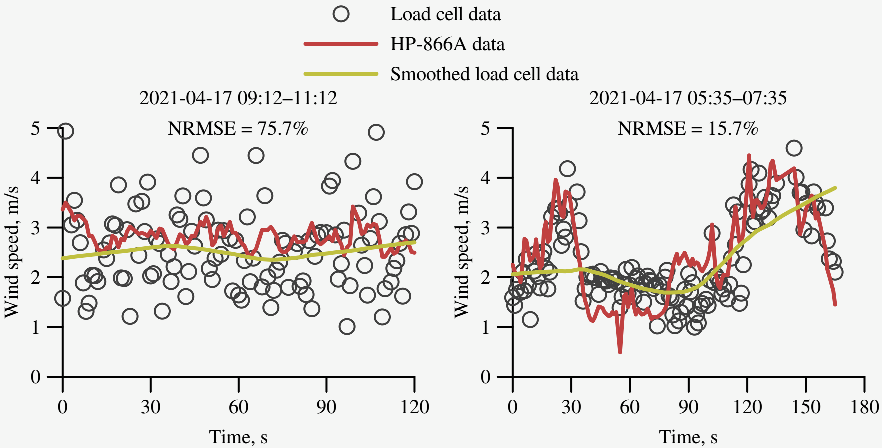
largest error



smallest error

J. Carta, C. Bueno, P. Ramírez *Statistical modelling of directional wind speeds using mixtures of von Mises distributions: Case study*, 2008.

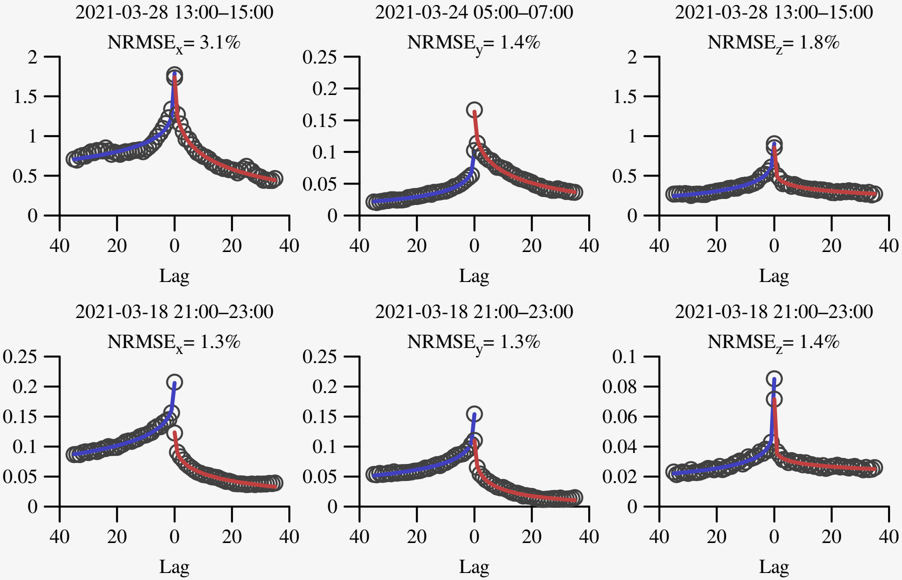
Verification of wind velocity against commercial anemometer



first site

second site

Verification of ACF against approximation

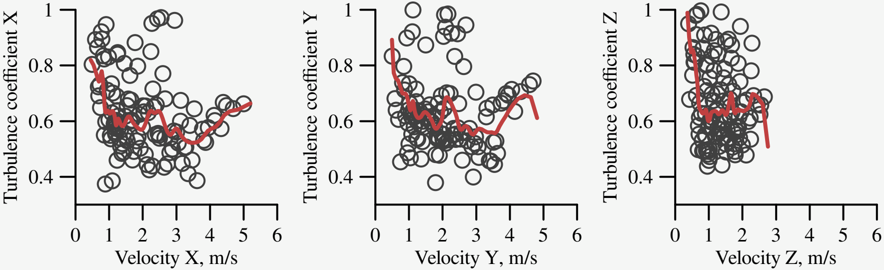


largest error

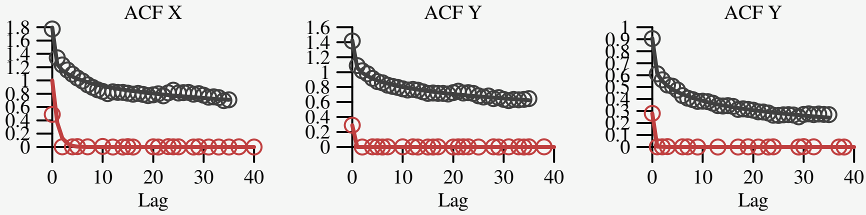
smallest error

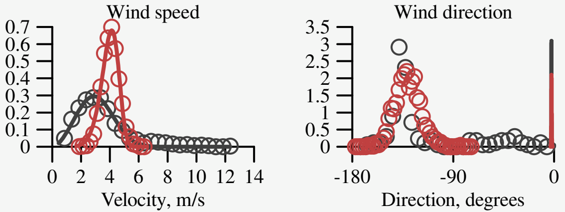
Turbulence coefficient

The ratio of absolute mean speed of turbulent flow to the absolute mean speed of incident flow for each axis.



Wind simulation





Model parameters:

	ACF a	ACF b	ACF c	Mean v, m/s
x	1.793	0.0214	0.2603	-2.439
y	1.423	0.01429	0.2852	-2.158
z	0.9075	0.06322	0.3349	-1.367

Conclusion and future work

- Per-axis wind speeds fit into Weibull distribution with max. NRMSE of 6.7%.
- Wind directions fit into von Mises distribution with max. NRMSE of 11%.
- Three-axis anemometer is useful for statistical studies, but is unable to measure *immediate* wind speed and direction.
- Simulated with autoregressive model and real wind velocity are similar in shape, but too far away from each other.

Future work: construct an array of anemometers to measure spatial autocovariance.